


UNIVERSITY OF
ILLINOIS LIBRARY
AT URBANA-CHAMPAIGN
BOOKSTACKS



Digitized by the Internet Archive
in 2011 with funding from
University of Illinois Urbana-Champaign

<http://www.archive.org/details/inflationcapital901leec>



BEER

FACULTY WORKING
PAPER NO. 901

THE LIBRARY OF THE

UIC 281-87

UNIVERSITY OF ILLINOIS
AT URBANA-CHAMPAIGN

Inflation and Capital Asset Pricing Determination: A Theoretical and Empirical Investigation

Cheng F. Lee
Frank C. Jen

College of Commerce and Business Administration
Bureau of Economic and Business Research
University of Illinois, Urbana-Champaign

BEBR

FACULTY WORKING PAPER No. 901

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

September 1982

Inflation and Capital Asset Pricing Determination:
A Theoretical and Empirical Investigation

Cheng F. Lee
IBE Distinguished Professor of Finance
Department of Finance

Frank C. Jen
State University of New York at Buffalo

Acknowledgment: The authors are grateful to the University of Illinois Research Board for its research assistant support, and to Carolyn Alt for her computer assistance and comments.

ABSTRACT

Based upon responsive coefficient concepts, this study has theoretically re-examined the impacts of inflation on capital asset pricing. The relationship between the real parameters of CAPM and the nominal parameters of CAPM is derived. A multi-index model with the change in purchasing power as an additional variable is derived to test whether the real parameters are significantly different from the nominal parameters.

Using the data of 464 securities to test the model derived in this paper, it is found that the estimated real parameters of CAPM are generally not significantly different from the estimated nominal parameters if the real risk free rates are assumed to be nonstochastic. However, the multi-index model derived in this paper does show the importance of inflation variable in estimating the related parameters in capital asset pricing determining process.

I. Introduction

Kennedy (1960), Brian (1969), Johnson, Reilly and Smith (1971), Hendershott and Van Horne (1973), Ondet (1973), Reilly, Smith and Johnson (1975) and others have shown that inflation significantly influences common stock values and bond rates. However, the capital asset pricing model (CAPM) developed by Sharpe (1964), Linter (1965) and Mossin (1966) (SML) does not take into account the impacts of inflation; and therefore, the parameters of SML type CAPM are generally estimated and interpreted in terms of the nominal rates of return without further justification.

Most recently, Roll (1973), Merton (1973), Long (1974), Chen and Bones (1975) and Hagerman and Kim (1976) (HK) have studied the possible impacts of inflation on the capital asset pricing model. However, they have not explicitly taken into account the degree of inflation hedge associated with the individual company (or industry). In testing the multi-period, two-parameter model, Fama and Macbeth (1974) (FM) have found that beta coefficients of the CAPM estimated by observed real rates of return are not statistically significantly different from those estimated by nominal rates. However, they provide no explanation of their empirical results. The main purposes of this study are to derive a CAPM that allows us to estimate all the parameters associated with the impacts of inflation on the capital asset pricing model; and to analyze the possible differences between the nominal and the real systematic risk and Jensen's measure of performance. Results of this study also are compared with those obtained by Roll (1973) and Hagerman and Kim (1976).

In the second section the CAPM with price level changes developed by Roll and HK is reviewed and criticized by using the concept of rates of return without inflation. In the third section, the relationship between real parameters of CAPM and nominal parameters of CAPM is investigated in accordance with the multivariate normal assumption. In the fourth section, a generalized CAPM is derived to allow the change of purchasing power to be an additional explanatory variable in the capital asset pricing model. Impacts of inflation on capital asset pricing also is analyzed in some detail. In the fifth section, data of 464 securities selected from NYSE are employed to test the impacts of inflation on both the estimated systematic risk and the estimated Jensen performance measure. The same data also are used to test the importance of including the change in purchasing power as an additional explanatory variable in capital asset pricing. Both the nominal and real Jensen measure, systematic risk, total risk and nonsystematic risk are also used to test the bias of the Jensen measure. Finally, results of this paper are summarized.

II. Real CAPM and Its Implications

To derive the CAPM in terms of real rates of return, Roll and HK have assumed that (i) all investors are risk averse single-period maximizers of expected utility of real terminal wealth and have quadratic utility functions; (ii) all investors act in terms of identical joint probability distributions of end-of-period outcomes; (iii) markets are perfect; (iv) there are no transaction costs and (v) borrowing and lending is risk free in nominal but not in real terms.

To review and criticize the real CAPM development by Roll and HK, three alternative rates of return concepts, i.e., rates of return without inflation, nominal rates of return, and observed real rates of return are used to explain the capital asset pricing process. The relationship among these three alternative rates of return is defined as

$$\begin{aligned}
 (a) \quad R_j^O &= R_j^T R_I^{e_j-1} \\
 (b) \quad R_m^O &= R_m^T R_I^{e_m-1} \\
 (c) \quad R_f^O &= R_f^T R_I^{e_f-1}
 \end{aligned} \tag{1}$$

where $R_j^O = 1 +$ observed real rates of return on the j^{th} asset.

$R_j^T = 1 +$ rates of return on j^{th} asset without inflation.

$R_m^O = 1 +$ observed real market rates of return.

$R_m^T = 1 +$ market rates of return without inflation.

$R_f^O = 1 +$ observed real rates of return on investor borrowing or lending.

$R_f^T = 1 +$ rates of return on investor borrowing and lending without inflation.

$R_I =$ one plus the change of general price level.

e_j , e_m and e_f are defined as inflation response coefficients which measure the impacts of inflation on rates of return on j^{th} risky asset, market rate of return, and borrowing (or lending) rate respectively.

$R_j^T R_I^{e_j}$, $R_m^T R_I^{e_m}$ and $R_f^T R_I^{e_f}$ are holding period nominal rates of return and R_j^O , R_m^O and R_f^O are observed real rates of return as defined by Roll and KH.

Equations (2A), (2B), and (2C) can be used to estimate the response coefficients, e_j , e_m , and e_f

$$\begin{aligned}
 (A) \quad \log_e R_{jt}^N &= \log_e R_j^T + e_j \log R_{It} + \varepsilon_{jt} \\
 (B) \quad \log_e R_{mt}^N &= \log_e R_m^T + e_m \log R_{It} + \varepsilon_{mt} \\
 (C) \quad \log_e R_{ft}^N &= \log_e R_f^T + e_f \log R_{It} + \varepsilon_{ft}
 \end{aligned} \tag{2}$$

where

$$R_{jt}^N = R_j^T R_{It}^{e_j} = 1 + \text{nominal rates of return for } j^{\text{th}} \text{ security}$$

$$R_{mt}^N = R_m^T R_{It}^{e_m} = 1 + \text{nominal market rates of return}$$

$$R_{ft}^N = R_f^T R_{It}^{e_f} = 1 + \text{nominal risk-free rate}$$

The estimated e_j , e_m , and e_f can be used to test whether $R_j^O = R_j^T$, $R_m^O = R_m^T$ and $R_f^O = R_f^T$ or not. If these estimated response coefficients are not significantly different from one, then observed real rates of return are not significantly different from real rates of return without inflation. These analyses and their implications will be explored further in the empirical sections (Section V).

Since the borrowing and lending rate is not risk-free in real term, Hagerman and Kim have derived a real CAPM as

$$E(\tilde{R}_j^O) = E(\tilde{R}_f^O) + \text{Cov}(\tilde{R}_m^O, \tilde{R}_j^O - \tilde{R}_f^O) / \text{Cov}(\tilde{R}_m^O, \tilde{R}_m^O - \tilde{R}_f^O) [E(\tilde{R}_m^O) - E(\tilde{R}_f^O)] \tag{3}$$

HK have shown that their real CAPM will reduce to Roll's real CAPM (p. 915, equation 26) when either R_f^O is nonstochastic or uncorrelated with the observed real market rate of return. Using the notations of (3), Roll's real CAPM is written as

$$E(\tilde{R}_j^O) = R_f^O + \text{Cov}(\tilde{R}_m^O, \tilde{R}_j^O) / \text{Var}(\tilde{R}_m^O) [E(\tilde{R}_m^O) - R_f^O] \tag{4}$$

Merton (1973) has shown that S-M-L type CAPM has implicitly assumed that the investment opportunity is constant over time. He also argued that the changes in the interest rate and purchasing power are main factors affecting the change in the investment opportunity set. These findings imply that rates of return without inflation rather than nominal rates of return should be used to test S-M-L type CAPM.¹ However, Roll, KH, FM and others have used the observed real rates of return rather than the rates of return without inflation to analyze as well as test the impacts of inflation on the capital asset pricing.

Next, the advantage of using rates of return without inflation rather than observed real rates of return concepts are analyzed. From equations (1) and (2), it is obvious that the observed real rates of return will not be equal to rates of return without inflation unless all of inflation response coefficients, e_j , e_m and e_f , are equal to one. If all response coefficients are equal to one, then the nominal rates of return can be expressed in terms of observed real rates of return and inflation rate as²

$$\begin{aligned}
 (a) \quad R_j^N &= R_j^O + \Delta I + \Delta I \cdot R_j^O \\
 (b) \quad R_m^N &= R_m^O + \Delta I + \Delta I \cdot R_m^O \\
 (c) \quad R_f^N &= R_f^O + \Delta I + \Delta I \cdot R_f^O
 \end{aligned}
 \tag{5}$$

where ΔI represents the inflation rate.

Gibson (1970), Jaffee and Mandelker (1976) and others have regarded these relationships as a summarization of price expectation (or Fisher effect). Fisher (1930), Fama (1975) and others have shown that the

relationship of (4c) approximately hold. However, the relationships of both (4a) and (4b) generally do not hold.³ Empirically, Jaffee and Mandelker (1976) have shown that the "Fisher effect" does not hold for market index.

Finally, one of HK's main results as indicated in (2) will be analyzed. Equation (3) implies that the slope of regressing $(\tilde{R}_j^o - \tilde{R}_j^o)$ on $(\tilde{R}_m^o - \tilde{R}_f^o)$ should be defined as

$$b_j = \frac{\text{Cov}(\tilde{R}_j^o, \tilde{R}_m^o) - \text{Cov}(\tilde{R}_m^o, \tilde{R}_f^o)}{\text{Var}(\tilde{R}_m^o) - \text{Cov}(\tilde{R}_m^o, \tilde{R}_f^o)} \quad (6)$$

If the real market return is uncorrelated with price level change, HK has shown that equation (6) can be rewritten as

$$\beta_j = \text{Cov}(\tilde{R}_j^o, \tilde{R}_m^o) / \text{Var}(\tilde{R}_m^o) \quad (6')$$

The difference between b_j and β_j can be written as

$$b_j = \beta_j - (1 - \beta_j)(c / (1 - c)) \quad (7)$$

where

$$c = \text{Cov}(\tilde{R}_m^o, \tilde{R}_f^o) / \text{Var}(\tilde{R}_m^o) .$$

c is the regression coefficient of regressing \tilde{R}_f^o on \tilde{R}_m^o ; it can be used to measure the relationship between observed real market rate of return associated with the observed real interest rate. Since Merton (1973) has shown that the change of interest rate is one of the important factors affecting the change of the investment opportunity set, it seems reasonable to conclude that the degree of stability of estimated b_j

instead of estimated β_j has taken shift of investment opportunity into account.

It can be argued that the observed real beta estimate β_j is only a proxy of the observed real estimate b_j and $(1-\beta_j)(c/1-c)$ is the related proxy error. Empirically, the estimated b_j can be indirectly obtained from the regressions defined as

$$\tilde{R}_{jt}^o - \tilde{R}_{ft}^o = \alpha_j + \beta_j (\tilde{R}_{mt}^o - \tilde{R}_f^o) + \epsilon_{jt}$$

$$\tilde{R}_{ft}^o = a + c \tilde{R}_{mt}^o + \tilde{I}_t$$

In the following section, the difference between the real parameters of CAPM and nominal parameters of CAPM is analyzed in a multivariate normal distribution framework.

III. Relationship Between Nominal and Real Parameters Under Multivariate Normal Assumption

If R_f^o is nonstochastic in equation (4), a regression relationship in terms of observed real rates of return is defined as

$$(\tilde{R}_{jt}\tilde{P}_t - R_f^o) = \alpha_j + \beta_j (\tilde{R}_{mt}\tilde{P}_t - R_f^o) + \epsilon_{jt} \quad (8)$$

Where \tilde{R}_j and \tilde{R}_m are nominal rates of return for j^{th} security and nominal market rate of return respectively, $\epsilon_{jt} \sim N(0, \sigma_j^2)$ and $\tilde{P}_t = R_I^{-1}$. From equation (7), it is obvious that the observed real systematic risk and Jensen performance measure can be defined as

$$(a) \quad \beta_j = \text{Cov}(\tilde{R}_{jt}\tilde{P}_t, \tilde{R}_{mt}\tilde{P}_t) / \text{Var}(\tilde{R}_{mt}\tilde{P}_t) \quad (9)$$

$$(b) \quad \alpha_j = E(\tilde{R}_{jt}\tilde{P}_t - R_f^o) - \beta_j [E(\tilde{R}_{mt}\tilde{P}_t) - R_f^o]$$

where $E(\cdot)$ is the expectation operator. By using the trivariate normal moment generating function and the definition of covariance, it can be shown that⁴

$$\begin{aligned}
 (a) \quad \text{Cov}(\tilde{R}_{jt}, \tilde{R}_{mt}) &= [\text{Cov}(\tilde{R}_{jt} \tilde{P}_t, \tilde{R}_{mt} \tilde{P}_t) - \text{Var}(\tilde{P}_t) E(\tilde{R}_{jt} \tilde{R}_{mt}) \\
 &\quad - \text{Cov}(\tilde{R}_{jt}, \tilde{P}_t) \text{Cov}(\tilde{R}_{mt}, \tilde{P}_t) - \overline{PR}_m \text{Cov}(\tilde{R}_{jt}, \tilde{P}_t) \\
 &\quad - \overline{PR}_j \text{Cov}(\tilde{R}_{mt}, \tilde{P}_t)] / \overline{P}^2
 \end{aligned} \tag{10}$$

$$\begin{aligned}
 (b) \quad \text{Var}(\tilde{R}_{mt}) &= [\text{Var}(\tilde{R}_{mt} \tilde{P}_t) - \text{Var}(\tilde{P}_t) E(\tilde{R}_{mt}^2) - \text{Cov}^2(\tilde{R}_{mt} \tilde{P}_t) \\
 &\quad - \alpha \overline{PR}_m \text{Cov}(\tilde{R}_{mt}, \tilde{P}_t)] / \overline{P}^2
 \end{aligned}$$

where $\overline{R}_m = E(\tilde{R}_{mt})$, $\overline{P} = E(\tilde{P}_t)$ and $\overline{R}_j = E(\tilde{R}_{jt})$

From (9a), (9b), (10a) and (10b), the relationship between real parameters and nominal parameters is defined as

$$\begin{aligned}
 (a) \quad \beta_j' &= \beta_j + S_j \\
 (b) \quad \alpha_j' &= \alpha_j / \overline{P} - E(\tilde{R}_{mt}) (\beta_j + S_j),
 \end{aligned} \tag{11}$$

where: β_j' = nominal systematic risk, α_j' = nominal Jensen performance measure, $S_j = \{ \text{Var}(\tilde{P}_t) [\beta_j E(\tilde{R}_{mt}^2) - E(\tilde{R}_{jt} \tilde{R}_{mt}) + \text{Cov}(\tilde{R}_{mt}, \tilde{P}_t)]$
 $[\beta_j \text{Cov}(\tilde{R}_{mt}, \tilde{P}_t) + 2\beta_j \overline{PR}_m - \text{Cov}(\tilde{R}_{jt}, \tilde{P}_t) - \overline{PR}_j] - \overline{PR}_m \text{Cov}(\tilde{R}_{jt}, \tilde{P}_t) \}$
 $/ [\text{Var}(\tilde{R}_{mt} \tilde{P}_t) - \text{Var}(\tilde{P}_t) E(\tilde{R}_{mt}^2) - \text{Cov}^2(\tilde{R}_{mt} \tilde{P}_t) - 2\overline{PR}_m \text{Cov}(\tilde{R}_{mt}, \tilde{P}_t)]$

If \tilde{P}_t is independent of both \tilde{R}_{jt} and \tilde{R}_{mt} , then equation (11a) is reduced to

$$\begin{aligned} \beta_j' &= \beta_j + \frac{\text{Var}(\tilde{P}_t)[\beta_j E(\tilde{R}_{mt}^2) - E(\tilde{R}_{jt}, \tilde{R}_{mt})]}{\text{Var}(\tilde{R}_{mt} \tilde{P}_t) - \text{Var}(\tilde{P}_t) E(\tilde{R}_{mt}^2)} \\ &= \beta_j - \frac{\text{Var}(\tilde{P}_t) E(\tilde{P}_t \tilde{R}_{mt}) (\alpha_j)}{E(\tilde{P}_t^2) [\text{Var}(\tilde{R}_{mt} \tilde{P}_t) - \text{Var}(\tilde{P}_t) E(\tilde{R}_{mt}^2)]^2} \end{aligned} \quad (12)$$

Under this circumstance, the condition of nominal systematic risk being reduced to the real systematic risk are: (i) inflation is certain, (ii) the real intercept is zero and (iii) the average real market rate of return is zero. Incidentally, these conclusions are similar to Casson's (1973) results in investigating the problem of linear regression with errors in the deflating variable. However, the assumption of independence between \tilde{P}_t and nominal rates of return is relatively strong; hence, other alternative assumptions are needed for investigating the relationship between real parameters and nominal parameters.⁵ In the following section, a multi-index model based upon our generalized assumptions as defined in equation (1) is derived and the impacts of inflation on capital asset pricing are investigated.

IV. A Model For Testing the Impacts of Inflation on Capital Asset Pricing

Using Taylor's expansion, we can linearize the nominal rates of return in terms of rates of return without inflation, R_I and response coefficients as ⁶

$$\begin{aligned} (a) \quad R_j^N &= R_j^T + e_j R_I \\ (b) \quad R_m^N &= R_m^T + e_m R_I \\ (c) \quad R_f^N &= R_f^T + e_f R_I \end{aligned} \quad (13)$$

If R_f^T is nonstochastic and equal to R_f^O , then a CAPM in terms of rates of return without inflation can be defined as

$$(R_{jt} - e_j R_{It} - R_f) = \alpha_j + \beta_j (R_{mt} - e_m R_{It} - R_f) + \epsilon_{jt} \quad (14)$$

The systematic risk and Jensen performance measure estimated from equation (14) are not necessarily free from the impacts of inflation. From equation (14) real systematic risk and real Jensen performance measure is defined as

$$(a) \quad \beta_j = [\text{Cov}(R_{jt}, R_{mt}) - e_m \text{Cov}(R_{jt}, R_{It}) - e_j \text{Cov}(R_{mt}, R_{It}) + (e_j e_m) \text{Var}(R_{It})] \\ / [\text{Var}(R_{mt}) + e_m^2 \text{Var}(R_{It}) - 2e_m \text{Cov}(R_{mt}, R_{It})] \quad (15)$$

$$(b) \quad \alpha_j = E(R_{jt} - R_{It} - R_f^O) - \beta_j E(R_{mt} - e_m R_{It} - R_f^O)$$

If we define $e_j = \text{Cov}(R_{jt}, R_{It}) / \text{Var}(R_{It})$

$$e_m = \text{Cov}(R_{mt}, R_{It}) / \text{Var}(R_{It})$$

then the relationship between nominal parameters and real parameters can be defined as

$$(a) \quad \beta_j' = \beta_j - (e_j - \beta_j e_m)(d - 2e_m) / (1te_m^2 d - 2e_m^2) \quad (16)$$

$$(b) \quad \alpha_j' = \alpha_j - (e_j - \beta_j e_m)[(d - 2e_m)(R_{mt} - e_m R_{It} - R_f^O) \\ / (1te_m^2 d - 2e_m^2) - R_{It}]$$

where $d = \text{Var}(R_{It}) / \text{Var}(R_{mt})$

β_j' is nominal systematic risk and α_j' is nominal Jensen performance measure. Equation (16a) implies that the nominal systematic risk is equal

to the real systematic risk when either (i) inflation is certain or (ii) $\beta_j = e_j/e_m$. Equation (16b) implies that the nominal Jensen performance measure will reduce to the real Jensen performance measure if and only if $\beta_j = e_j/e_m$. Now a model derived from equation (14) is proposed to test whether β_j is equal to e_j/e_m as

$$(R_{jt} - R_f) = \alpha_j + \beta_j[R_{mt} - R_f] + (e_j - e_m\beta_j)R_{It} + \epsilon_{jt} \quad (17)$$

Equation (17) can be used to empirically test the impact of inflation on the systematic risk estimate. From the specification analysis, it is well-known that the regression coefficient associated with R_{It} can be decomposed into two components, i.e.,

(i) the regression coefficient of regressing R_{jt} on R_{It}

and

(ii) the auxiliary regression coefficient of regressing R_{mt} on R_{It} times the systematic risk ($e_m\beta_j$). If e_j is equal to $e_m\beta_j$, then the systematic risk obtained from nominal rates of return will not be different from the estimate obtained from the real rates of return. In other words, if the ratio among every risky asset's response coefficient (e_j) is equal to the ratios among every risky asset's systematic risk (β_j), then inflation will not affect the capital market pricing and therefore, the portfolio efficient frontier constructed from nominal rates of return will not be different from that obtained from the real rates of return.⁷

The method derived in this section is different from the last section in two important aspects, i.e.,

- (i) an additive rather than a multiplicative assumption is used to investigate the impact of inflation on capital asset pricing

and

(ii) the response coefficients (e_j and e_m) are introduced to take care of consequences of anticipated inflation on different companies' rates of return. Reilly (1975) has classified companies into (i) complete inflation hedge companies, (ii) superior inflation hedge companies and (iii) inferior inflation hedge companies. Our response coefficients, e_j , e_m and e_f defined in equation (1) can be used explicitly as a criterion for classifying companies according to degree of inflation hedge.

Now, our equation (17) is compared with either Roll's equation (27) or HK's equation (16).⁸ In terms of our notations, Roll's equation (27) can be defined as

$$E(\tilde{R}_j) = R_f^0 + [E(\tilde{R}_m - R_f^0)]\beta_j - [\text{Cov}(\tilde{R}_j, \tilde{P}) - \beta_j \text{Cov}(\tilde{R}_m, \tilde{P})]/E(\tilde{P}) \quad (18)$$

Equation (18) implies that the coefficient associated with $E(\tilde{R}_I)$ will be zero if

$$\beta_j = \text{Cov}(\tilde{R}_j, \tilde{R}_{It}) / \text{Cov}(\tilde{R}_m, \tilde{R}_{It}) = e_j / e_m \quad (19)$$

The implication of this equation is similar to our implication described in equation (17). Hence, the results of this section have given Roll's results a useful empirical interpretation. In the following section, data from NYSE are used to test both equations (8) and (17).

V. Some Empirical Results

For investigating the impacts of inflation on the estimated systematic risk and Jensen performance measure, 464 monthly individual stock

rates of return selected from the NYSE during the period of January 1966 to March 1979 are employed to estimate the β_j , β'_j , α_j and α'_j .⁹ The Fisher index with dividends is used to calculate the market rate of return, the monthly 90 days treasury bill rate is used as a measurement of risk free rate, and the monthly consumer price index is used to measure inflation rate in estimating the real systematic risk and Jensen performance measure. The results of averaged systematic risk and the Jensen performance measure in both nominal terms and real terms for two sub-periods are listed in Columns 1 and 2 of Table I respectively. The first sub-period is from January 1966-December 1972 and the second sub-period is from January 1973-December 1979.

The results indicate that the estimated real parameters are not significantly different from the estimated nominal parameters. To test the specification of equation (17), 464 multiple regressions are estimated by using the data described in this section. The regression coefficient associated with R_{It} in equation (17) can be used to test whether the estimated β_j is significantly different from estimated e_j/e_m or not.

Results are listed in Table II-a. All significant estimated \hat{a}_2 's have negative sign. These imply that inflation generally has negative impacts on the security rates of return. To obtain some further insights about the impacts of inflation on security rates of return, Equation (2) is estimated by using 464 individual firms' data and the data of market rates of return. The results associated with individual firms all listed in Table II-b. It is also found that the inflation has significant negative impact on the market rates of return. Our empirical results indicate that the inflation will generally have some effects on the

magnitude of estimated systematic risk and Jensen's measure of performance. This conclusion is essentially based upon the fact that some of the ratios between estimated response coefficient for the j^{th} security and the response coefficient for the market portfolio is significantly different from the estimated systematic risk for j^{th} security.

Friend and Blume (1970) have regressed the performance measures on both total risk and systematic risk to determine the possible bias associated with performance measure estimates. We now use the models indicated in equations (20-21) to do further empirical analysis.

$$\hat{\alpha}_j = a_0 + a_1 \hat{\beta}_j + \epsilon_j \quad (20)$$

$$\hat{\alpha}_j = b_0 + b_1 \sigma_j^2 + \epsilon_j \quad (21)$$

where α_j is the estimate Jensen measure for j^{th} security; $\hat{\beta}_j$ is the estimated systematic risk for j^{th} security and σ_j^2 is the total risk for j^{th} security. Empirical results for two-subperiods are listed in Tables II-a and II-b. For the first subperiod, the Jensen measure obtained from real rates of return, which obtained by $\frac{R}{1+r}$, does not show the bias as found by Friend and Blume (1970). However, the results of second subperiod do show the bias which was found by Friend and Blume.

VI. Summary

Based upon responsive coefficient concepts, this study has theoretically re-examined the impacts of inflation on capital asset pricing. The relationship between the real parameters of CAPM and the nominal parameters of CAPM is derived. A multi-index model with the change in purchasing power as an additional variable is derived to test whether

the real parameters are significantly different from the nominal parameters.

Using the data of 464 securities to test the model derived in this paper, it is found that the estimated real parameters of CAPM are generally not significantly different from the estimated nominal parameters if the real risk free rates are assumed to be nonstochastic. However, the multi-index model derived in this paper does show the importance of inflation variable in estimating the related parameters in capital asset pricing determining process.

FOOTNOTES

¹Merton (1980) has argued that the ex ante instead of the ex post measure should be used to do the theoretical test. Rate of return without inflation is an ex ante measure and observed real rate of return is an ex post measure.

²See Fama (1975) or Jaffee and Mandelker (1976, p. 455).

³See section IV for detail.

⁴See appendix A.

⁵Based upon the assumption that real market return is uncorrelated with the price level change, HK (equation on 14b) have also derived the relationship between nominal systematic risk and systematic risk. However, they do not obtain the interesting implications as developed in this section.

⁶Chen and Bones (1975, p. 471) have used this kind of technique to derive some similar relationships. In addition, these results are similar to those defined in equation (2).

⁷Biger (1976) has found that there exist some differences between the efficient portfolio frontier obtained from real rates of return and those obtained from nominal rates of return.

⁸It can be shown that Roll's equation (27) is identical to HK's equation (16).

⁹The specification of (8) with and without adjusting for the purchasing power change is used to estimate the related parameters.

APPENDIX (A)

The derivation of (10):

If V_{jt} , V_{mt} and P_t are trivariate normally distributed, then from Hogg and Craig (1969, Chapter 13), the moment generating function of this trivariate normal distribution can be written as:

$$\begin{aligned} (c) \quad (t_1, t_2, t_3) = & \exp(t_1 \bar{R}_j + t_2 \bar{R}_m + t_3 \bar{P} + 1/2[t_1^2 \text{Var}(R_{jt}) + t_2^2 \text{Var}(R_{mt}) \\ & + t_3^2 \text{Var}(P_t) + 2t_1 t_2 \text{Cov}(R_{jt}, R_{mt}) + 2\text{Cov}(R_{jt}, P_t) \\ & + 2\text{Cov}(R_{mt}, P_t)]) \end{aligned}$$

From (c), it can be shown that

$$\begin{aligned} (d) \quad E(R_{jt} R_{mt} P_t^2) &= \frac{\partial^{(4)}(t_1, t_2, t_3)}{\partial t_1 \partial t_2 \partial t_3^2} \quad t_1 = t_2 = t_3 = 0 \\ &= \text{Cov}(R_{jt}, R_{mt}) \sigma_P^2 + \sigma_P^2 \bar{R}_j \bar{R}_m + \bar{P}^2 \text{Cov}(R_{jt}, R_{mt}) \\ &\quad + \bar{R}_j \bar{P} \text{Cov}(R_{mt}, P_t) + \bar{R}_m \bar{P} \text{Cov}(R_{jt}, P_t) + 2\text{Cov}(R_{mt}, P_t) \\ &\quad \text{Cov}(R_{jt}, P_t) + \bar{P}^2 \bar{R}_j \bar{R}_m + \bar{R}_m \bar{P} \text{Cov}(R_{jt}, P_t) + \bar{R}_j \bar{P} \text{Cov}(R_{mt}, P_t) \end{aligned}$$

From the definition of covariance, we also have

$$(e) \quad E(R_{jt} P_t) = \text{Cov}(R_{jt}, P_t) + \bar{R}_j \bar{P}$$

$$(f) \quad E(R_{mt} P_t) = \text{Cov}(R_{mt}, P_t) + \bar{R}_m \bar{P}$$

From the moment generating function of bi-variate normal distribution, it can be shown that

$$\begin{aligned}
 (g) \quad E(R_{mt}^2 P_t^2) &= \frac{\partial^{(4)}(t_1, t_2)}{\partial^2 t_1 \partial^2 t_2} & t_1 = t_2 = 0 \\
 &= \sigma_m^2 \sigma_p^2 + \sigma_m^2 \overline{P}^2 + \sigma_p^2 \overline{R}_m^2 + 2\text{Cov}^2(R_{mt}, P_t) \\
 &\quad + 2\overline{R}_m \overline{P} \text{Cov}(R_{mt}, P_t) + \overline{R}_m^2 \overline{P}^2
 \end{aligned}$$

Substituting (d), (e) and (f) into (b), then we have (10b), and substituting (f) and (g) into (a), then we have (10a).

REFERENCES

1. Biger, N. (1976), "Portfolio Selection and Purchasing Power Risk--Recent Canadian Experiences," Journal of Financial and Quantitative Analysis, 11, pp. 251-267.
2. Brodie, Z. (1976), "Common Stocks as Hedge Against Inflation," Journal of Finance, 31, pp. 459-470.
3. Brian, M. (1969), "Inflation and Common Stock Values," Journal of Finance, 24, pp. 530-535.
4. Casson, M. C. (1973), "Linear Regression With Error in the Deflating Variable," Econometrica, 41, pp. 751-759.
5. Chen, A. H. and J. A. Boness (1975), "Effects of Uncertain Inflation on Investment and Financing Decisions of a Firm," Journal of Finance, 30, pp. 469-483.
6. Fama, E. F. (1975), "Short-term Interest Rates as Predictors of Inflation," American Economic Review, 59, pp. 269-282.
7. Fama, E. F. and J. Macbeth (1974), Tests of Multiperiod Two-Parameter Models," Journal of Financial Economics, 1, pp. 607-636.
8. Friend, I. and M. Blume (1970), "Measurement of Portfolio Performance Under Uncertainty," American Economic Review, 60, pp. 561-575.
9. Gibson, W. E. (1970), "Price Expectations Effects on Interest Rates," Journal of Finance, 25, pp. 19-34.
10. Fisher, I. (1930), The Theory of Interest, New York.
11. Hagerman, R. L. and E. H. Kim (1976), "Capital Asset Pricing With Price Level Change," Journal of Financial and Quantitative Analysis, forthcoming.
12. Hendershott, P. H. and J. C. Van Horne (1973), "Expected Inflation Implied by Capital Market Rates," Journal of Finance, 28, pp. 301-314.
13. Hogg, R. V. and A. T. Craig (1969), Introduction to Mathematical Statistics, New York: MacMillan Co.
14. Jaffe, J. F. and G. Mandelker (1976), "The 'Fisher Effect' for Risky Assets: An Empirical Investigation," Journal of Finance, 31, pp. 447-458.
15. Johnson, Glenn L., Frank K. Reilly and Ralph E. Smith (1971), "Individual Common Stocks as Inflation Hedges," Journal of Financial and Quantitative Analysis, 6, pp. 1015-1024.

16. Kennedy, Charles (1960), "Inflation and the Bond Rate," Oxford Economic Papers, 12, pp. 269-274.
17. Lintner, J. (1965), "Security Prices, Risk, and Maximal Gains from Diversification," Journal of Finance, 20, pp. 587-616.
18. Long, J. B., Jr. (1974), "Stock Prices, Inflation, and the Term Structure," Journal of Financial Economics, 1, pp. 131-170.
19. Merton, R. C. (1973), "An Intertemporal Capital Asset Pricing Model," Econometrica, 41, (September), pp. 867-887.
20. Merton, R. C. (1980), "On Estimating Expected Return on the Market: An Exploratory Investigation," Journal of Financial Economics, 8, pp. 323-361.
21. Mossin, J. (1966), "Equilibrium in a Capital Asset Market," Econometrica, 34, pp. 768-783.
22. Ondet, B. A. (1973), "The Variation of the Return on Stocks in Periods of Inflation," Journal of Financial Quantitative Analysis, pp. 247-258.
23. Reilly, F. K. (1975), Companies and Common Stocks as Inflation Hedges, Center for the Study of Financial Institutions, New York University (Bulletin, 1975-2).
24. Reilly, Frank K., Ralph E. Smith and Glenn L. Johnson (1975), "A Correction and Update Regarding Individual Common Stocks as Inflation Hedges," Journal of Financial and Quantitative Analysis, 10 pp. 871-880.
25. Reilly, F. K. and G. L. Johnson (1970), "Inflation, Inflation Hedges and Common Stock," Financial Analysts Journal, (January-February), pp. 104-110.
26. Mossin, J. (1966), "Equilibrium in a Capital Asset Market," Econometrica, 34, pp. 768-783.
27. Ondet, B. A. (1973), "The Variation of the Return on Stocks in Periods of Inflation," Journal of Financial Quantitative Analysis, pp. 247-258.
28. Roll, Richard (1973), "Assets, Money, and Commodity Price Inflation Under Uncertainty," Journal of Money, Credit and Banking.
29. Sharpe, W. F. (1964), "Capital Asset Prices: A Theory of Market Equilibrium Under Condition of Risk," Journal of Finance, 19, pp. 425-442.

Table I

AVERAGE REAL AND NOMINAL PARAMETERS*

			Real Value	Nominal Value
Systematic Risk	1966-72	Division	1.10362 (.41543)	1.10291 (.41417)
		Subtraction	1.10467 (.41328)	
	1973-79	Division	1.06219 (.34490)	1.0614 (.34448)
		Subtraction	1.06158 (.33901)	
Jensen Performance Measure	1966-72	Division	-.00262 (.00727)	-.00223 (.00738)
		Subtraction	-.00187 (.00774)	
	1973-79	Division	.00090 (.00891)	.00093 (.00898)
		Subtraction	.00137 (.00997)	

*Standard errors appear in parentheses beneath the corresponding coefficients.

Table II-a

Percentage Significant of a_2 in $R_{jt} - R_{ft} = a_0 + a_1(R_{mt} - R_{ft}) + a_2R_{It} + e_{jt}$

PERIOD	5% LEVEL		10% LEVEL	
	No.	%	No.	%
1966-72	73	16%	127	27%
1973-79	42	9%	84	18%

Table II-b

Percentage Significance of a_1 in $R_{jt} = a_0 + a_1$ in $R_{It} + e_{jt}$

	5% LEVEL		10% LEVEL	
	No.	%	No.	%
1966-72	109	23%	159	34%
1973-79	90	19%	151	33%

Table III-a

REGRESSION RESULTS 1966-1972

Model 1: $\hat{\alpha}_j = a_0 + a_1 \epsilon_j + \epsilon_j$

	a_0	a_1	Unadj. R^2	SEE
Nominal term	-.00542** (.0096)	.00289** (.00082)	.02630	.00730
Real term	-.00153 (.00096)	.00098 (.00081)	.00315	.00728

Model 2: $\hat{\alpha}_j = b_0 + b_1 \epsilon_j^2 + \epsilon_j$

	b_0	b_1	Unadj. R^2	SEE
Nominal term	-.00446** (.00058)	.29794** (.06255)	.04680	.00722
Real term	-.00311** (.00058)	.06592 (.06360)	.00232	.00728

**Significant at .01 level

*Significant at .05 level

Table III-b

REGRESSION RESULTS 1973-1979

Model 1: $\alpha_j = a_0 + a_1 \beta_j + \epsilon_j$

	a_0	a_1	Unadj. R^2	SEE
Nominal term	-.00768** (.00129)	.00812** (.00115)	.09694	.00856
Real term	-.00762** (.00128)	.00805** (.00114)	.09711	.00849

Model 2: $\alpha_j = b_0 + b_1 \sigma_j^2 + \epsilon_j$

	b_0	b_1	Unadj. R^2	SEE
Nominal term	-.00539** (.00063)	.65844** (.00540)	.24347	.00783
Real term	-.00534** (.00628)	.66223** (.05432)	.24343	.00770

**Significant at .01 level

*Significant at .05 level

ECKMAN
INDERY INC.



JUN 95

nd To Please N MANCHESTER.
INDIANA 46962

UNIVERSITY OF ILLINOIS-URBANA



3 0112 060296156